Objective # 43  Square Roots and Operations with Radicals

Material:  Teacher Notes

Homework:  worksheet

Parts of a Radical:

\[ \sqrt[n]{x} \]

- **Square Root:** is a radical operation (index = 2) performed on a number to get a value, such that the value when squared will give the original number back.

- **Cube Root:** is a radical operation (index = 3) performed on a number to get a value, such that the value when cubed will give the original number back.

Example:  Perform the following operations:

a) \[ \sqrt{36} = 6 \]  

b) \[ \sqrt{64} = 8 \]  

c) \[ \sqrt{169} = 13 \]  

d) \[ \frac{3}{8} = 2 \]  

e) \[ \frac{3}{27} = 3 \]

_Simplest Form of a Square Root:_ A square root is in its simplest form when the radicand does NOT contain a factor which is a perfect square.

_Perfect Squares:_ 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, etc.

Example:  Simplify each of the following:

a) \[ \sqrt{50} \] is not in its simplest form because 50 has a factor of 25 which is a perfect square

\[ \sqrt{50} = (\sqrt{25})(\sqrt{2}) = (5)(\sqrt{2}) = 5\sqrt{2} \]

b) \[ \sqrt{8} = (\sqrt{4})(\sqrt{2}) = (2)(\sqrt{2}) = 2\sqrt{2} \]

c) \[ \sqrt{48} = (\sqrt{16})(\sqrt{3}) = (4)(\sqrt{3}) = 4\sqrt{3} \]

d) \[ \sqrt{80} = (\sqrt{16})(\sqrt{5}) = (4)(\sqrt{5}) = 4\sqrt{5} \]

e) \[ \sqrt{288} = (\sqrt{4})(\sqrt{72}) = (\sqrt{4})(\sqrt{9})(\sqrt{8}) = (\sqrt{4})(\sqrt{9})(\sqrt{4})(\sqrt{2}) \]

\[ = (2)(3)(2)(\sqrt{2}) = 12\sqrt{2} \]

f) \[ \sqrt{288} = (\sqrt{144})(\sqrt{2}) = (12)(\sqrt{2}) = 12\sqrt{2} \]

g) \[ 7\sqrt{175} = (7)(\sqrt{25})(\sqrt{7}) = (7)(5)(\sqrt{7}) = 35\sqrt{7} \]

_NOW DO EXERCISE 1 ON THE WORKSHEET!_
**Multiplying and Dividing Square Roots:** If two square roots are being multiplied or divided, then write a single square root and move the multiplication or division inside that square root.

Example: Perform the following operations and simplify all radicals:

a) \( (\sqrt{2})(\sqrt{3}) = \sqrt{(2)(3)} = \sqrt{6} \)

b) \( \frac{\sqrt{24}}{\sqrt{8}} = \sqrt{\frac{24}{8}} = \sqrt{3} \)

c) \( (3\sqrt{5})(4\sqrt{10}) = (3)(4)(\sqrt{5})(\sqrt{10}) = 12\sqrt{50} = (12)(\sqrt{25})(\sqrt{2}) = (12)(5)(\sqrt{2}) = 60\sqrt{2} \)

d) \( \frac{32\sqrt{54}}{2\sqrt{3}} = \left( \frac{32}{2} \right) \left( \frac{\sqrt{54}}{\sqrt{3}} \right) = 16\sqrt{18} = (16)(\sqrt{9})(\sqrt{2}) = (16)(3)(\sqrt{2}) = 48\sqrt{2} \)

*NOW DO EXERCISE 2 ON THE WORKSHEET!*

**Like Square Roots:** Like Square Roots are square roots that have the same number inside the radical sign.

Examples: Like Square Roots: \( \sqrt{3}, \quad 4\sqrt{3}, \quad -2\sqrt{3} \)

NOT Like Square Roots: \( \sqrt{3}, \quad 4\sqrt{5}, \quad -2\sqrt{7} \)

**Adding and Subtracting Square Roots:** ONLY like square roots can be added or subtracted ... carry the common square root and add or subtract the coefficients (Numbers in front of the square root)

Example: Perform the following operations and simplify all radicals:

a) \( 3\sqrt{5} - 7\sqrt{5} + 8\sqrt{5} = 4\sqrt{5} \)

b) \( \sqrt{50} + \sqrt{18} \) **NOTE:** These are not like square roots and can’t be added but they can be simplified

\[
(\sqrt{25})(\sqrt{2}) + (\sqrt{9})(\sqrt{2}) = 5\sqrt{2} + 3\sqrt{2} = 8\sqrt{2}
\]

Once they are simplified we notice that they are now like square roots and can now be added

\[
3\sqrt{28} - \sqrt{63} = (3)(\sqrt{4})(\sqrt{7}) - (\sqrt{9})(\sqrt{7})
\]

\[
(\sqrt{2} + \sqrt{3})(\sqrt{3} + \sqrt{2})
\]

c) \( = (3)(2)(\sqrt{7}) - (3)(\sqrt{7}) = 6\sqrt{7} - 3\sqrt{7} = 3\sqrt{7} \)

\[
(\sqrt{2} + \sqrt{3})(\sqrt{3} + \sqrt{2})
\]

d) \( \sqrt{6} - \sqrt{4} + \sqrt{9} + \sqrt{6} \)

\[
2\sqrt{6} - 2 + 3 = 2\sqrt{6} + 1
\]

*NOW DO EXERCISE 3 and 4 ON THE WORKSHEET!*
Square Roots and Operations with Radicals

Worksheet

1. Express each of the following radicals in their simplest form:
   a) $\sqrt{12}$
   b) $\sqrt{20}$
   c) $\sqrt{18}$
   d) $\sqrt{27}$
   e) $\sqrt{98}$
   f) $\sqrt{72}$
   g) $\sqrt{125}$
   h) $\sqrt{396}$
   i) $\sqrt{363}$
   j) $2\sqrt{44}$
   k) $7\sqrt{128}$
   l) $4\sqrt{300}$

2. Perform the following operations and simplify all radicals:
   a) $(\sqrt{2})(\sqrt{5})$
   b) $(3\sqrt{2})(\sqrt{6})$
   c) $(\sqrt{8})(\sqrt{6})$
   d) $\frac{\sqrt{72}}{\sqrt{6}}$
   e) $\frac{\sqrt{50}}{\sqrt{5}}$
   f) $\frac{27\sqrt{490}}{9\sqrt{5}}$
   g) $\left(\sqrt{8}\right)\left(\frac{1}{\sqrt{2}}\right)$
   h) $(2\sqrt{15})(3\sqrt{30})$
   i) $(6\sqrt{2})(6\sqrt{18})$
   j) $\left(\frac{2}{\sqrt{5}}\right)\left(\frac{9}{\sqrt{2}}\right)\left(\frac{10}{\sqrt{3}}\right)$
   k) $\frac{\sqrt{7}}{\sqrt{63}}$
   l) $\frac{24\sqrt{56}}{6\sqrt{7}}$
3. Perform the following operations and simplify all radicals:

a) \( 3\sqrt{5} + 4\sqrt{5} \)  
b) \( 2\sqrt{7} + 7\sqrt{2} \)  
c) \( 14\sqrt{8} - 5\sqrt{8} \)

d) \( 2\sqrt{11} + 7\sqrt{11} - 4\sqrt{11} \)  
e) \( 7\sqrt{6} + 4\sqrt{3} - 3\sqrt{6} + 2\sqrt{2} \)

f) \( \sqrt{8} + \sqrt{18} \)  
g) \( \sqrt{75} - \sqrt{20} \)  
h) \( \sqrt{27} + \sqrt{48} - 2\sqrt{3} \)

i) \( -5\sqrt{44} + 2\sqrt{99} \)  
j) \( 3\sqrt{72} + 2\sqrt{75} - 3\sqrt{27} + \sqrt{108} \)

h) \( \sqrt{250} - \sqrt{135} - \sqrt{40} + \sqrt{735} \)

4. Perform the following operations and simplify all radicals:

a) \( (\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3}) \)  
b) \( (\sqrt{6} + \sqrt{2})^2 \)

c) \( (2\sqrt{5} + \sqrt{2})(\sqrt{3} + \sqrt{6}) \)  
d) \( (\sqrt{2} + \sqrt{6})(\sqrt{10} - \sqrt{3}) \)

e) \( (\sqrt{8} - \sqrt{6})^2 \)